Dissipative acoustic solitons

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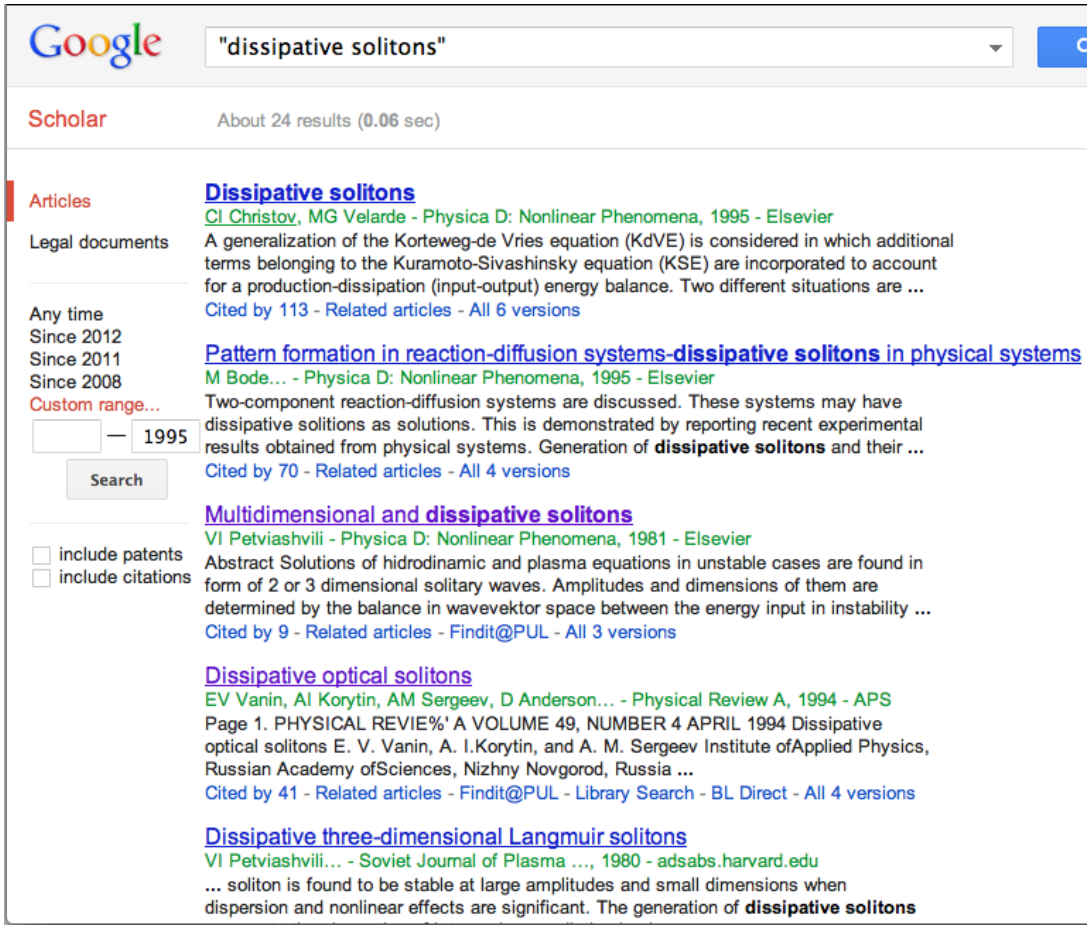
In Memoriam



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Dissipative solitons
CI Christov, MG Velarde - Physica D: Nonlinear Phenomena, 1995 - Elsevier
A generalization of the Korteweg-de Vries equation (KdVE) is considered in which additional terms belonging to the Kuramoto-Sivashinsky equation (KSE) are incorporated to account for a production-dissipation (input-output) energy balance. Two different situations are ...
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Pattern formation in reaction-diffusion systems-**dissipative solitons** in physical systems
M Bode... - Physica D: Nonlinear Phenomena, 1995 - Elsevier
Two-component reaction-diffusion systems are discussed. These systems may have dissipative solitions as solutions. This is demonstrated by reporting recent experimental results obtained from physical systems. Generation of **dissipative solitons** and their ...
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Multidimensional and **dissipative solitons**
VI Petviashvili - Physica D: Nonlinear Phenomena, 1981 - Elsevier
Abstract Solutions of hidrodinamic and plasma equations in unstable cases are found in form of 2 or 3 dimensional solitary waves. Amplitudes and dimensions of them are determined by the balance in wavevektor space between the energy input in instability ...
Cited by 9 - Related articles - Findit@PUL - All 3 versions

Dissipative optical solitons
EV Vanin, AI Korytin, AM Sergeev, D Anderson... - Physical Review A, 1994 - APS
Page 1. PHYSICAL REVIE%' A VOLUME 49, NUMBER 4 APRIL 1994 Dissipative optical solitons E. V. Vanin, A. I.Korytin, and A. M. Sergeev Institute ofApplied Physics, Russian Academy ofSciences, Nizhny Novgorod, Russia ...
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Dissipative three-dimensional Langmuir solitons
VI Petviashvili... - Soviet Journal of Plasma ..., 1980 - adsabs.harvard.edu
... soliton is found to be stable at large amplitudes and small dimensions when dispersion and nonlinear effects are significant. The generation of **dissipative solitons**

Christo Ivanov Christov (1951 – 2012)

`http://christov.metacontinuum.com`

The big picture

- Motivation:
  - Effect of fluctuations upon mean flow of a compressible fluid?
    - Average PDE $\rightarrow$ coarse-grained eq. (RANS & LES in turbulence).
    - Average the variational principle $\rightarrow$ "nonlinearly dispersive" corrections.
  - Implications for acoustics models? (Canonical problems.)
  - Vanishing-viscosity *or* vanishing-dispersion limit from NS to Euler?

- Outline of the talk:
  1. Derivation of weakly-nonlinear Lagrangian-averaged Euler-α (LAE-α) model.
  2. "Dispersive Taylor shocks" under the linearized eq. of motion (EoM).
  3. Energy analysis and exact traveling wave solutions (TWSs) of the nonlinear problem.
  4. Numerical simulations of TWS interactions using scheme with internal iterations.(Dissipative solitons.)

Background

- α-models of turbulence (Foias, Holm, Titi, Guermond, et al., ca. 1999–2003).
  - Introduce energy penalty in NS to prevent creation of excitations below a minimal length scale, α.
- For incompressible fluids, α can also represent:
  - the cell size in a finite-scale Lagrangian numerical simulation (Margolin, *TRSA*, 2009),
  - the first material modulus of a second-grade non-Newtonian fluids (formal analogy).
- For compressible fluids, α is the flow-dependent small-scale amplitude fluctuations over which the Lagrangian mean is taken.
- Bhat & Fetecau (*DCDS-B*, 2006): LAE-α model of compressible flow for perfect gases (i.e., $\wp \propto \varrho^\gamma$).
- We follow Lesser & Seebass (*JFM*, 1968): finite-amplitude, aka weakly-nonlinear, approach to eq. of motion for general $\wp(\varrho)$.

Eqs. of motion for 1D homentropic flow of an inviscid fluid

- Mass, momentum and energy conservation:

$$\varrho_t + (\varrho u)_x = 0,$$
$$\varrho(u_t + u u_x) + \wp_x = 0\alpha^2[\varrho(u_{xt} + u u_{xx})]_x,$$
$$\eta_t = \eta_t + u\eta_x = 0,$$

where $u = \nabla\phi = \big(u(x,t), 0, 0\big)$.

- Close system w/ quadratic approximation to barotropic eq. of state

$$\wp - \wp_{\mathrm{e}} = \varrho_{\mathrm{e}} c_{\mathrm{e}}^2 \left[\frac{\varrho - \varrho_{\mathrm{e}}}{\varrho_{\mathrm{e}}} + (\beta - 1)\left(\frac{\varrho - \varrho_{\mathrm{e}}}{\varrho_{\mathrm{e}}}\right)^2 \right],$$

- Introduce dimensionless variables/quantities

$$s = (\varrho - \varrho_{\mathrm{e}})/\varrho_{\mathrm{e}}, \quad u^\diamond = u/V, \quad \phi^\diamond = \phi/(LV),$$
$$x^\diamond = x/L, \quad t^\diamond = t(c_{\mathrm{e}}/L), \quad \underbrace{\epsilon = V/c_{\mathrm{e}}}_{\text{Mach number}}, \quad \underbrace{a = \alpha/L}_{\text{Knudsen number}}.$$

Weakly-nonlinear theory and reduction to a scalar equation

- To justify continuum equation with averaged fluctuations, $a \ll 1$ (small Knudsen number).
- Assume weak nonlinearity $\epsilon \ll 1$ and w.l.o.g. $a^2 = \mathcal{O}(\epsilon)$.
- After some work...

$$\boxed{\phi_{xx} - \phi_{tt} + a^2 \phi_{ttxx} = \epsilon \left[2(\beta - 1)\phi_t \phi_{xx} + \partial_t (\phi_x)^2 \right],}$$

  a new weakly-nonlinear dispersive inviscid acoustic equation.
- Distinguished limits:
  - $a \to 0$: recover straightforward weakly-nonlinear lossless acoustic wave equation (see I.C.C., C.I.C. & Jordan, *QJMAM*, 2007).
  - $\epsilon \to 0$: recover van Wijngaarden equation for sound waves in bubbly liquids (also Love's equation from classical elasticity).

Linearization: Love's equation

1. Set $\epsilon = 0$ to get $\phi_{xx} - \phi_{tt} + a^2\phi_{ttxx} = 0$.
2. The pressure satisfies the same eq., consider the signaling IBVP:

$$\begin{aligned} &p_{xx} - p_{tt} + a^2 p_{ttxx} = 0, && (x,t) \in (0,\infty)\times(0,\infty), \\ &p(0,t) = H(t), \quad p(\infty,t) = 0, && t \in (0,\infty), \\ &p(x,0) = 0, \quad p_t(x,0) = 0, && x \in (0,\infty). \end{aligned}$$

3. Solve by Laplace transform in t to get
$$p(x,t) = H(t)\left[1 - \frac{2}{\pi}\int_0^{1/a} \frac{\cos(\eta t)}{\eta} \sin\left(\frac{\eta x}{\sqrt{1-a^2\eta^2}}\right) \mathrm{d}\eta\right].$$
4. Or, solve by Fourier sine transform in x to get
$$p(x,t) = H(t)\left[1 - \frac{2}{\pi}\int_0^{\infty} \frac{\sin(\varsigma x)}{\varsigma(1+a^2\varsigma^2)} \cos\left(\frac{\varsigma t}{\sqrt{1+a^2\varsigma^2}}\right) \mathrm{d}\varsigma\right].$$
5. Notice $\eta = \varsigma/\sqrt{1+a^2\varsigma^2}$, similar to Stokes' first problem for second-grade fluid (I.C.C. & C.I.C., *Acta Mech.*, 2010).

Solution to the signaling IBVP for Love's equation

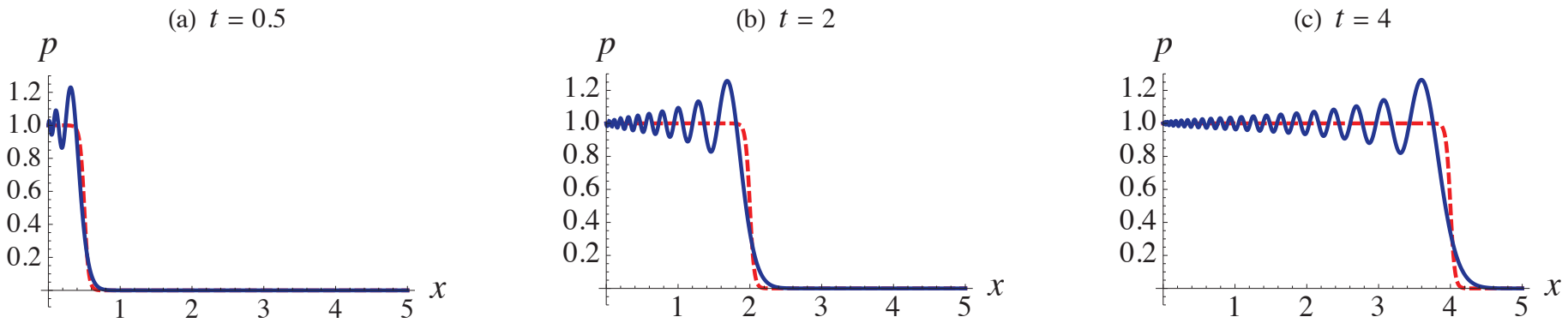


Figure : The solution profile p vs. x of IBVP (solid) compared to the Taylor shock profile $\frac{1}{2}\{1 - \tanh[(x-t)/(2a)]\}$ (dashed); $a = 0.03$.

Notice, EoM is linear yet

1. there's strong dispersive effects even for $a \ll 1$,
2. there's strong coherence of the wavefront.

...reminiscent of zero-dispersion limit of KdV eq. (e.g., Grava & Klein, *CPAM*, 2007).

Energy analysis

- Recall our EoM: $\phi_{xx} - \phi_{tt} + a^2\phi_{ttxx} = \epsilon\left[2(\beta-1)\phi_t\phi_{xx} + \partial_t(\phi_x)^2\right]$.
- Assume localized solution ($\lim_{|x|\to\infty}\phi_x, \phi_t \to 0$) and integrate over x:

$$\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}t} = -2\epsilon(\beta - 3/2)\int_{-\infty}^{+\infty}(\phi_t)^2\phi_{xx}\,\mathrm{d}x.$$

- Energy is given by

$$\mathcal{E} := \frac{1}{2}\int_{-\infty}^{+\infty} \underbrace{(\phi_x)^2}_{\text{kinetic energy}} + \underbrace{(\phi_t)^2}_{\text{potential energy}} + \underbrace{a^2(\phi_{xt})^2}_{\text{rotational kinetic energy at microscale}} \mathrm{d}x.$$

- Non-conservative nature of EoM entirely due to the nonlinearity.
- Sign of $\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}t}$ is not definite; for $\beta = \frac{3}{2}$, $\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}t} = 0$.
- For $\beta \neq \frac{3}{2}$, $\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}t} = 0$ only for certain distinguished solutions, i.e., dissipative solitons (as in C.I.C. & Velarde, *Physica D*, 1995).

Traveling wave solutions

- EoM invariant under $x \mapsto -x$, consider only right-traveling waves: $\phi(x,t) = F(\xi)$ with $\xi := x - ct$.
- EoM is reduced to an ODE for $f = F'$:

$$3a^2c^2(f')^2 + 3(1-c^2)f^2 + 2\epsilon\beta c f^3 = 6\mathfrak{K}_1 f + \mathfrak{K}_2.$$

- For supersonic wave ($c > 1$) with $\mathfrak{K}_1 = \mathfrak{K}_2 = 0$:

$$f(\xi) = A_0 \operatorname{sech}^2\left[\frac{(\xi-\xi_0)\sqrt{c^2-1}}{2ac}\right], \qquad A_0 = \frac{3(c^2-1)}{2c\epsilon\beta},$$

- In terms of the potential ϕ:

$$F(\xi) = \frac{2acA_0}{\sqrt{c^2-1}} \tanh\left[\frac{(\xi-\xi_0)\sqrt{c^2-1}}{2ac}\right],$$

which is a topological soliton or kink.

Finite-difference scheme with internal iterations

- Introduce auxiliary function $\psi := \phi_t$, EoM becomes

$$\phi_{tt} = \phi_{xx} + a^2\phi_{ttxx} - \epsilon[2(\beta-1)\psi\phi_{xx} + 2\phi_x\psi_x].$$

- Use three-time-level, linearly-implicit discretization:

$$\delta_{t+}\delta_{t-}\Phi_j^n = \delta_{x+}\delta_{x-}\left[\tfrac{1}{4}(\Phi_j^{n+1} + 2\Phi_j^n + \Phi_j^{n-1})\right] + a^2\delta_{t+}\delta_{t-}\delta_{x+}\delta_{x-}\Phi_j^n - \epsilon\left[2(\beta-1)\Psi_j^n\delta_{x+}\delta_{x-}\Phi_j^n + 2\delta_{x0}\Phi_j^n\delta_{x0}\Psi_j^n\right],$$

where $\Phi_j^n \approx \phi(x_j, t^n)$ and $\Psi_j^n \approx \psi(x_j, t^n)$.

- Internal iterations (e.g., C.I.C. & Velarde, *IJBC*, 1994) on k, guess $\Psi_j^{n,0} = \delta_{t-}\Phi_j^n$, find Φ_j^{n+1} from difference eq., then $\Psi_j^{n,k+1} = \delta_{t0}\Phi_j^n$.
- Convergence criterion $\max_j |\Psi_j^{n,k+1} - \Psi_j^{n,k}| < 10^{-8} \max_j |\Psi_j^{n,k}|$ is met within 3 to 10 internal iterations.

Simulation of soliton–anti-soliton interaction, $\beta < 3/2$

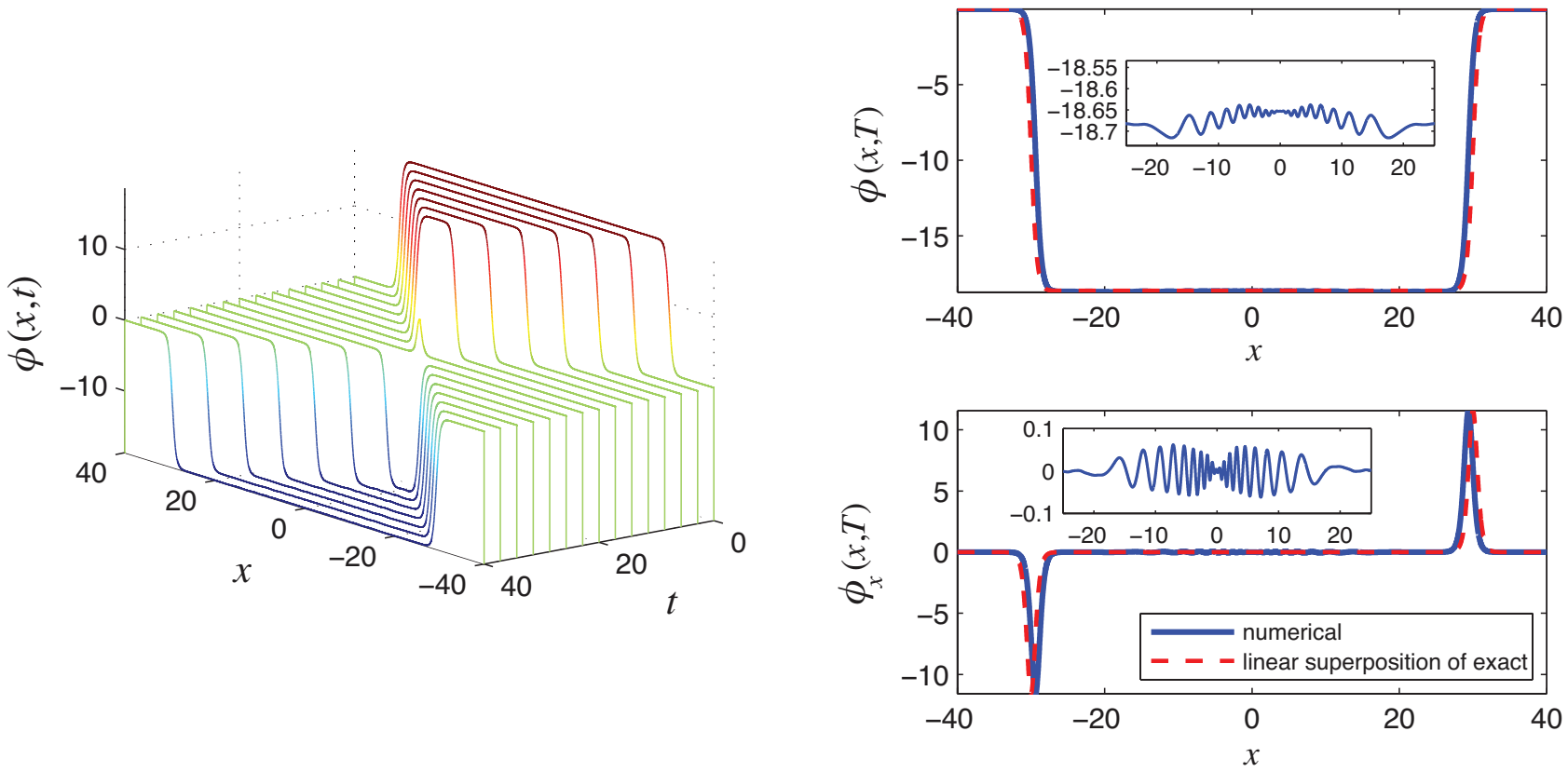


Figure : $\beta = 1.2$, $c_{1,2} = \pm 1.5$, $\xi_{0,\{1,2\}} = \mp 30$ and $T = 40$: space-time plot (left) and comparison of the post-interaction solitons with a linear superposition of the exact shapes (right). Clearly, the two solitons collide and re-emerge, retaining their shape ("identity"), save for a phase shift and the emission of small-amplitude radiation.

Simulation of soliton–soliton interaction, $\beta < 3/2$

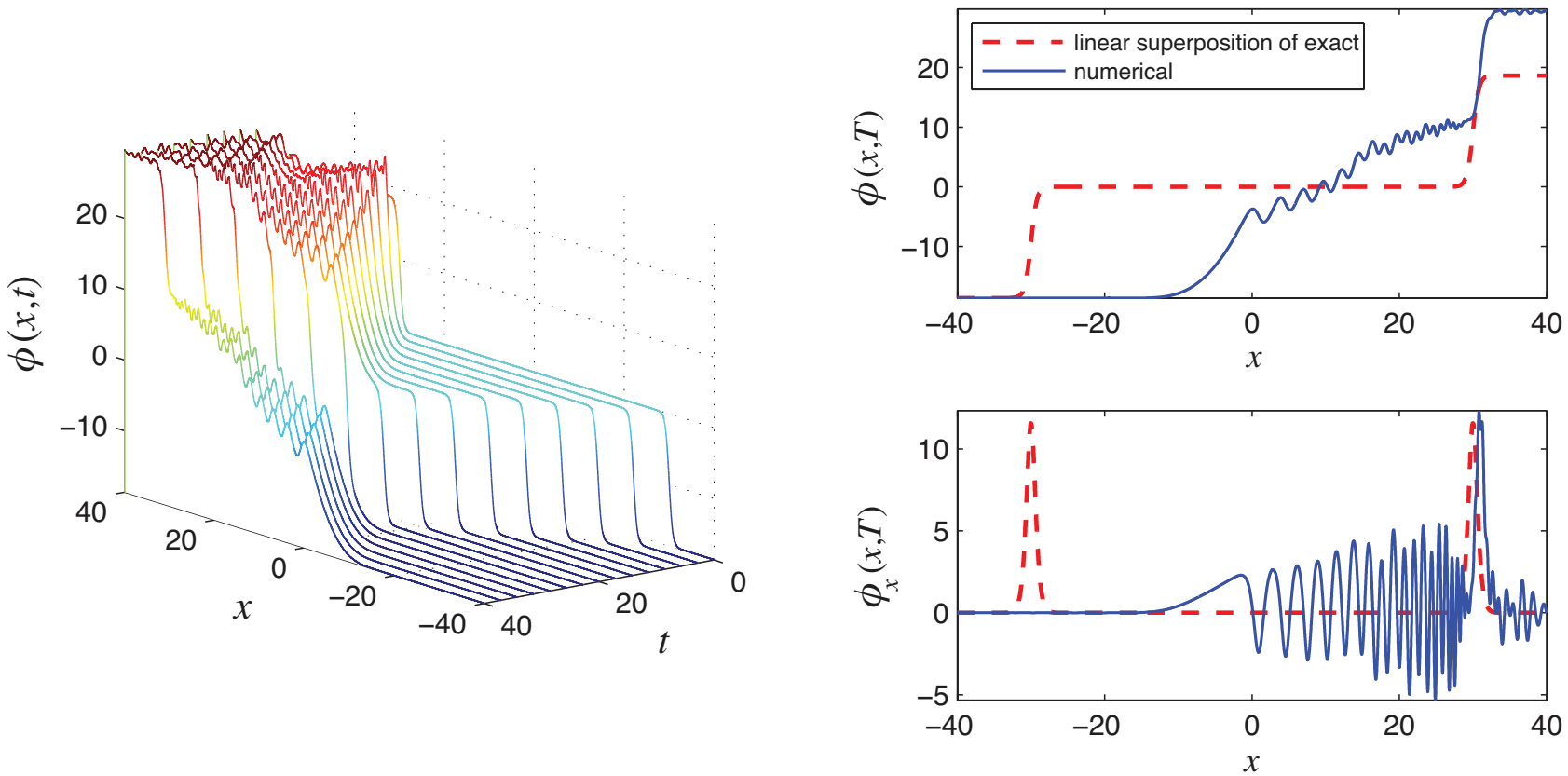
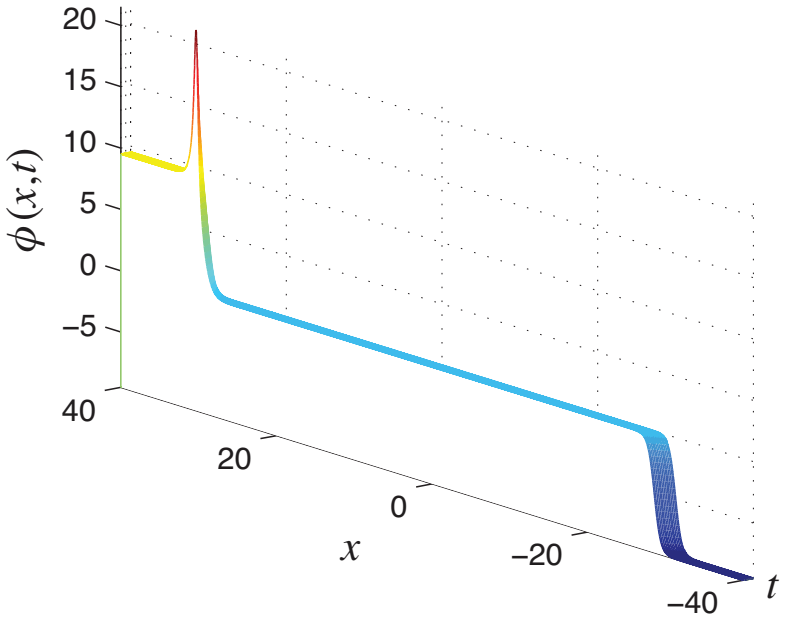


Figure : $\beta = 1.2$, $c_{1,2} = \pm 1.5$, $\xi_{0,\{1,2\}} = \mp 30$ and $T = 40$: space-time plot (left) and comparison of the post-interaction solitons with a linear superposition of the exact shapes (right). The right soliton disintegrates almost immediately, and so does the left one upon reaching the remains of the right soliton.

Simulation of soliton–soliton interaction, $\beta > 3/2$



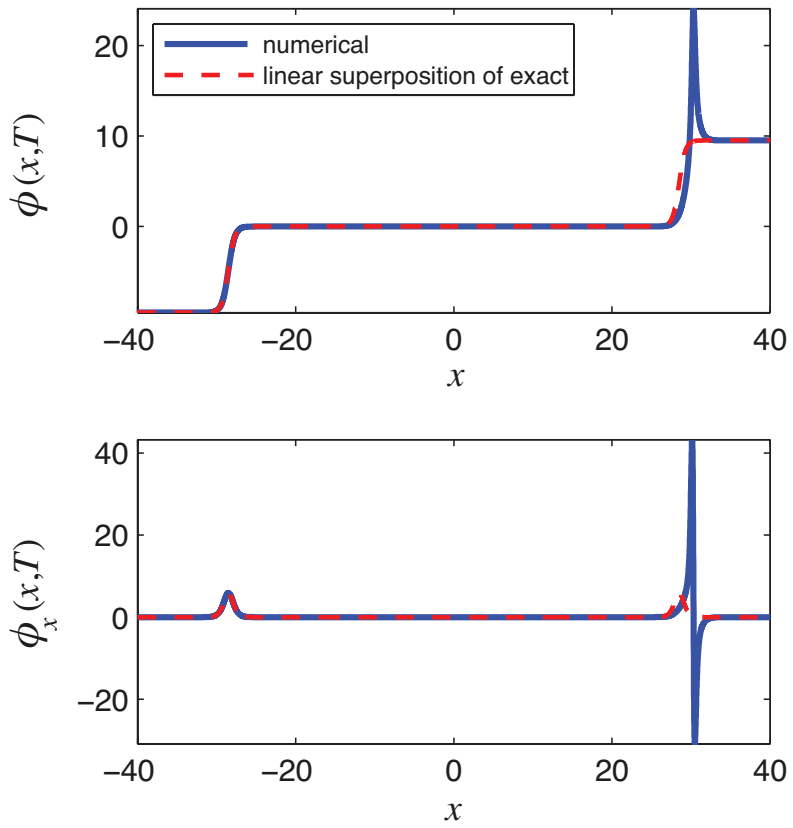


Figure : $\beta = 2.35$, $c_{1,2} = \pm 1.5$, $\xi_{0,\{1,2\}} = \mp 30$ and $T = 1$: space-time plot (left) and comparison of the post-interaction solitons with a linear superposition of the exact shapes (right). The right soliton develops a "horn" that grows without bound.

Summary & conclusions

- What we have found:
  1. A new 1D weakly-nonlinear LAE-α model eq. valid for gases & compressible liquids.
  2. Kink TWSs can retain their identity after a collision.
  3. Kink TWS behave as dissipative solitons.
  4. Unstable combinations of kinks that disintegrate even before impact.
  5. Instability can be explosive. [Proof?]
- What we have yet to do:
  1. Investigate of the stability of the kink solutions. [Proof?]
  2. Check whether $\exists$ ranges of the wave speed c for which the soliton–anti-soliton interaction leads to a bound state ("breathers").
  3. Can other weakly-nonlinear models (Rassmusen, Sørensen, et al., 2008–2011) support dissipative solitons?
  4. Add dissipation (e.g., viscosity, heat conduction, ...).

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