ME 69700NWM
NONLINEAR WAVE MECHANICS

Course Outcomes

1. Understand the physics of *dispersive* linear and nonlinear wave phenomena in continuum mechanics.
2. Learn *analytical methods* for linear equations (Fourier transform) and certain classes nonlinear wave equations (e.g., Cole–Hopf transform, traveling wave solution).
3. Become familiar with basic *numerical methods* to solve nonlinear wave problems for which analytical methods fail.
4. Acquire *unified* understanding of nonlinear waves mechanics by identifying universal features in fluids, solids, acoustics, metamaterials (+ other special topics).

Fundamental Concepts (1.0 wks)

1. Introduction: What is a wave?
2. Dispersion relations, phase velocity vs. group velocity.
3. Elementary 1D equations: waves along a discrete chain, waves along a continuous string.

Nonlinear Waves in Fluids (3.0 wks)

1. Acoustic waves in fluids.
2. The Burgers equation: a toy model with nonlinearity and dissipation.
3. The Korteweg–de Vries (KdV) equation: nonlinearity and dispersion, surface waves.

Analytical Methods (3.0 wks)

1. Solution to linear problems via Fourier transform.
2. Solution to nonlinear problems via special transformations.
3. Solution to nonlinear problems via traveling wave ansatz.

Nonlinear Waves in Solids (3.0 wks)

1. KdV equation in solids.
2. Coupled pendula and sine–Gordon equation.
3. Hertzian chain and "sonic bullets".
4. More chains: interaction forces, energy transfer.

Numerical Methods (2.0 wks)

1. Semi-discretization via finite difference and/or psudospectral methods.
2. Time stepping and stability issues.

Representative Projects

1. Simulate the KdV equation and demonstrate the collision property of solitons.
2. Study waves in a discrete chain, demonstrating the impact of pair-interaction forces.
3. Report on other types of nonlinear waves, such as kinks or breathers and their context.
4. Derive a nonlinear wave equation in a new context arising in the student's research.
5. Reproduce a key result from a recent paper in the literature.

Advanced Topics (3.0 wks)

1. Nonlinear waves in fluids with microstructure.
2. Nonlinear waves in metamaterials.
3. Degenerate nonlinear dispersion.
4. KdV–Burgers, other advanced equations.

COURSE NUMBER: ME 69700NWM

COURSE TITLE: Nonlinear Wave Mechanics

REQUIRED COURSE OR ELECTIVE COURSE: Elective

TERMS OFFERED: Fall

RECOMMENDED TEXTBOOK:

Remoissenet, M., *Waves Called Solitons*, 3rd ed., Springer-Verlag, 1999.

Billingham, J. & King, A. C., *Wave Motion*, Cambridge Univ. Press, 2000.

Drazin, P.G. & Johnson, R.S., *Solitons: An Introduction*, Cambridge Univ. Press, 1989.

RECOMMENDED: MA 52700, MA 52800, or equivalent knowledge
ME 50900, or equivalent knowledge
ME 58000, or equivalent knowledge
ME 58100, or equivalent knowledge

COORDINATING FACULTY: I. C. Christov

COURSE DESCRIPTION: Graduate-level introduction to nonlinear dispersive waves. Examples from classical continuum mechanics (fluids and solids), as well as frontier research fields (e.g., metamaterials). Basic analytical and numerical solution techniques to study nonlinear wave mechanics and interactions. Introduction to the current literature and research problems.

ASSESSMENTS TOOLS:
1. Bi-weekly problem sets.
2. Final project report and presentation.

PROFESSIONAL COMPONENT:
1. Engineering Topics: Engineering Science – 100%
   Engineering Design – 0%

NATURE OF DESIGN CONTENT: None.

COMPUTER USAGE: Knowledge of basic scientific programming (e.g., MATLAB, Python or equivalent) will be necessary.

COURSE STRUCTURE/SCHEDULE:
a. Lecture – 2 days per week at 75 minutes.

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RELATED ME PROGRAM OUTCOMES:
1. Engineering fundamentals
2. Engineering design
3. Communication skills
4. Ethical/Prof. responsibilities
5. Teamwork skills
6. Experimental skills
7. Knowledge acquisition

PREPARED BY: I. C. Christov

REVISION DATE: Sept. 17, 2020