Multiphysics problems at low Reynolds number

From deformable channels to spinning droplets

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120 SMALL STEPS TO ENGINEERING EXCELLENCE

Research Disciplines

[Fluid Mechanics] [Soft Matter] [Nonlinear Dynamics]

[Wave Propagation] [Applied Mathematics]

Education

- Feynman Fellow, CNLS, Los Alamos National Laboratory
- NSF Fellow, MAE, Princeton
- PhD in Applied Mathematics (2011), Northwestern
- MS in Mathematics (2007), Texas A&M
- SB in Mathematics (Applied Option) (2005), MIT

Mission Statement

I combine advanced mathematics with state-of-the-art simulation to model flowing materials (fluids, solids, gases, or anything in between), rationalize experiments, and make progress on ***fundamental* (basic scientific)** questions at the interface of engineering, mathematics, and physics. Specifically, an overarching theme in my research is **transport** (e.g., as a means of effecting mixing or for mitigating separation).

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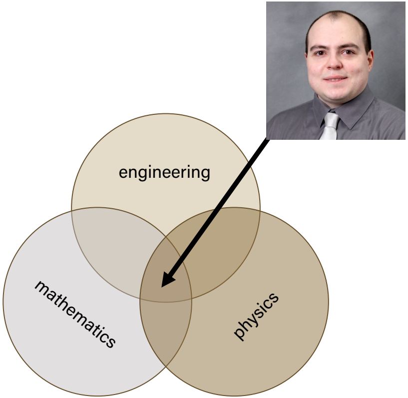

Outline: Our main objectives today

1. Rigorous, predictive theory of flow-induced deformation of 3D compliant microchannels.

2. Demonstrate shape and motion control of ferrofluid droplets by external magnetic fields.

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What do you picture when you hear "hydraulics"?

Figure: Industrial pipe network, `https://www.steeljrv.com/`

▷ Channel diameters ∼ meters, flow ∼ turbulent, materials hard ∼ steel.

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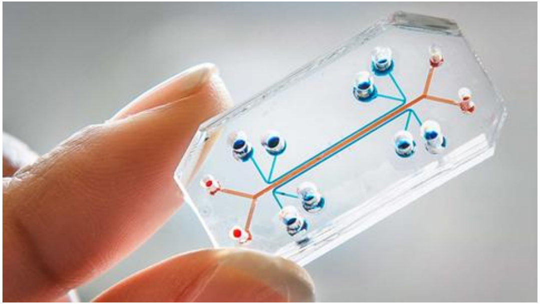


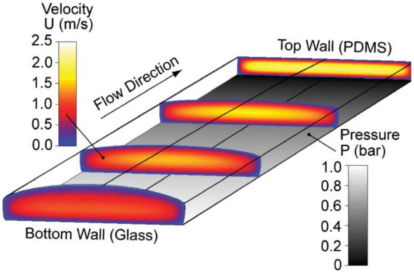
Figure: Microfluidic chip for chemical analysis, https://darwin-microfluidics.com/

▷ Channel diameters ∼ 100s μm, flow ∼ laminar, materials soft ∼ PDMS (gel-like).

Flow-induced deformation of compliant microchannels

- Microchannels fabricated from PDMS are soft ⇒ deform due to flow.

(Gervais *et al.*, *Lab Chip*, 2006)



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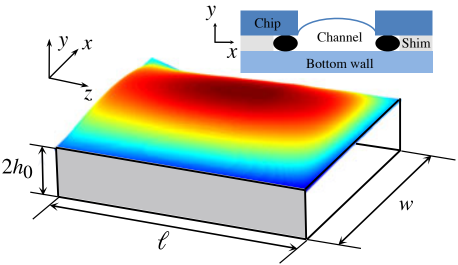

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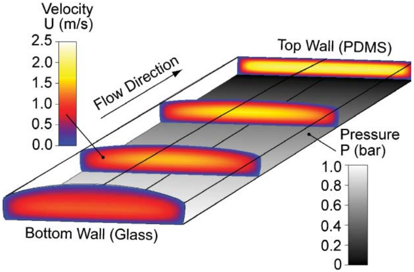

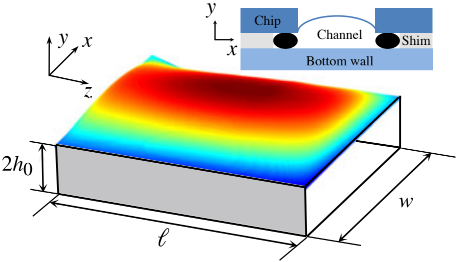

- For a rigid conduit, $q = \underbrace{R_h^{-1}}_{h_0^3 w / 12\mu\ell} \Delta p$ ("Poiseuille's law").

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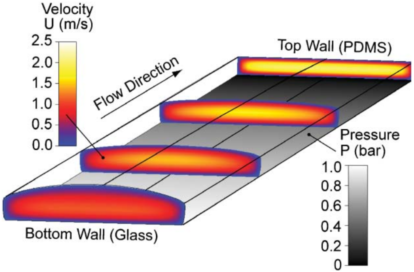

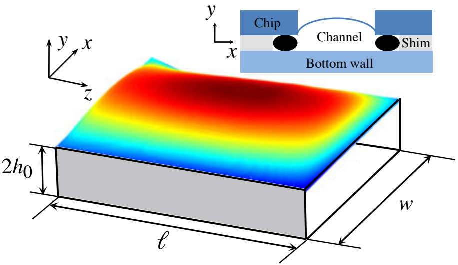

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- FSI $\Rightarrow$ need deformation–pressure for pressure–flow relations: $\boxed{\text{soft hydraulics problem}}$.

Building blocks: deformation–pressure relations

- 2D planar, vertical deform.: $u_y(z) = t\frac{p(z)}{2G+\lambda}$

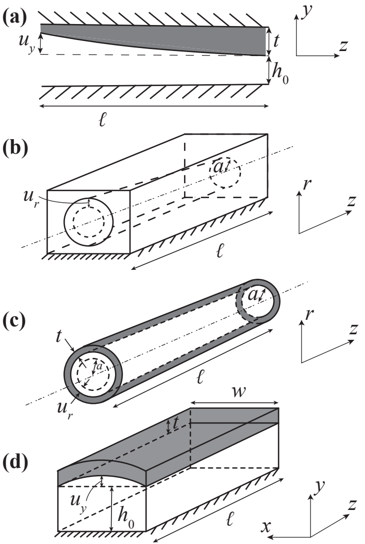
  (*e.g.*, Mukherjee *et al.*, *Soft Matter*, 2013)

- 3D axisym. inclusion, radial deformation: $u_r(z) = \frac{a}{4}\frac{p(z)}{G}$

  (*e.g.*, Raj M *et al.*, *Biomicrofluidics*, 2018)

- 3D axisym. shell, radial deformation: $u_r(z) = \frac{a^2}{t}\frac{p(z)}{\overline{E}}$, $\overline{E} = E/(1-\nu_s^2)$

  (*e.g.*, Anand & Christov, *ZAMM*, 2020)



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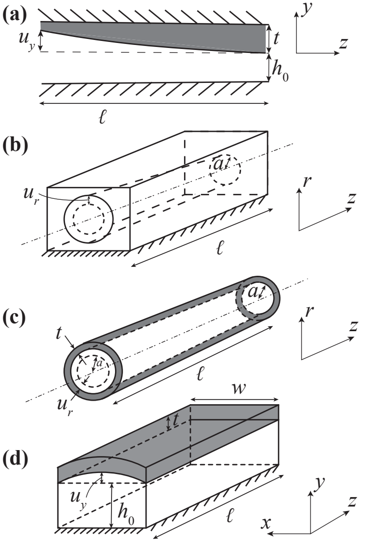
- 3D Cartesian, vertical deformation, first try: $\langle u_y \rangle_x(z) = \alpha\frac{p(z)}{\overline{E}}$.
  (Gervais *et al.*, *Lab Chip*, 2006)
- 3D Cartesian, vertical deformation, our approach: $u_y(x,z) = F(x)\frac{p(z)}{\overline{E}}$.
  (Christov *et al.*, *JFM*, 2018; Shidhore & Christov, *J. Phys.: Condens. Matt.*, 2018; Wang & Christov, *Proc. R. Soc. A*, 2019; Anand *et al.*, *J. Appl. Mech.*, 2020)



The nonlinear flow rate–pressure drop relation

- By definition of flow rate (capital letters = dimensionless):

$$\underbrace{1}_{q/q} = \int_{-1/2}^{+1/2} \underbrace{\int_0^{H(X,Z)} V_Z(X,Y,Z)\,\mathrm{d}Y}_{\text{lubrication theory}}\,\mathrm{d}X = \underbrace{-\frac{1}{12}\frac{\mathrm{d}P}{\mathrm{d}Z}\int_{-1/2}^{+1/2} [1 + \overbrace{\beta}^{\mathcal{U}_c/h_0} U_Y(X,Z)]^3\,\mathrm{d}X}_{\text{elasticity theory}}\,.$$

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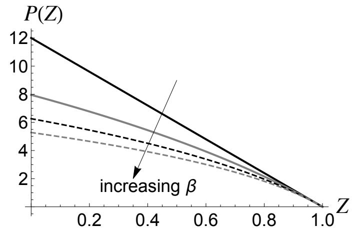
- Integrate ODE for $P(Z)$ subject to $P(1) = 0$ (outlet gauge p):

$$\boxed{12(1-Z) = P(Z)\left[1 + S_1\beta P(Z) + S_2\beta^2 P(Z)^2 + S_3\beta^3 P(Z)^3\right].}$$

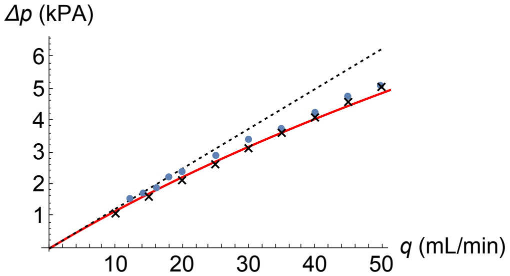
- S_i – constants found from X-variation of U_Y.
- No fitting or calibration, just (known) dimensionless group:

$$\beta \propto \frac{\mathcal{P}_c}{E}\mathfrak{F}(\text{geometry}).$$

(Christov *et al.*, *J. Fluid Mech.*, 2018)



Pressure drop: Experiment, numerics & theory

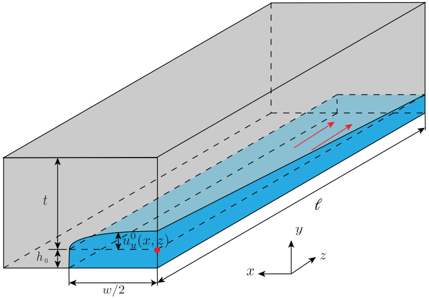


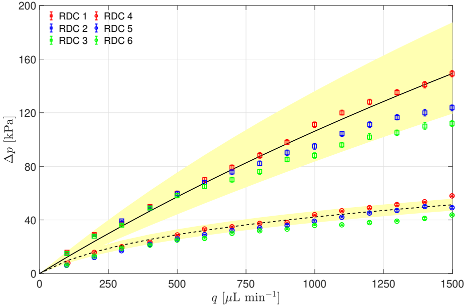
- •s: "S4" data (Ozsun *et al.*, *JFM*, 2013); ×s: our FSI simulation (Shidhore & Christov, *J. Phys.: Condens. Matt.*, 2018).
- Dashed line: rectangular conduit ($\sim$ Poiseuille), $\Delta p = \frac{12\mu\ell}{h_0^3 w} q$.
- Solid curve is our perturbative theory:

$$\Delta p = \underbrace{\frac{12\mu\ell}{h_0^3 w}\left[1 + \frac{1}{480}\frac{w^4\Delta p}{Bh_0} + \frac{1}{362,880}\left(\frac{w^4\Delta p}{Bh_0}\right)^2 + \frac{1}{664,215,552}\left(\frac{w^4\Delta p}{Bh_0}\right)^3\right]^{-1}}_{R_h^{\text{rigid}}\,\mathfrak{g}(\Delta p)} q.$$

Beyond plate theory: Thick structures

- Thin structure (plate-like, previous slide): $u_{\max} \ll t \sim h_0 \ll w \ll \ell$.
- Thick structure (half-space-like, this slide): $u_{\max} \ll h_0 \ll w \ll t \ll \ell$.

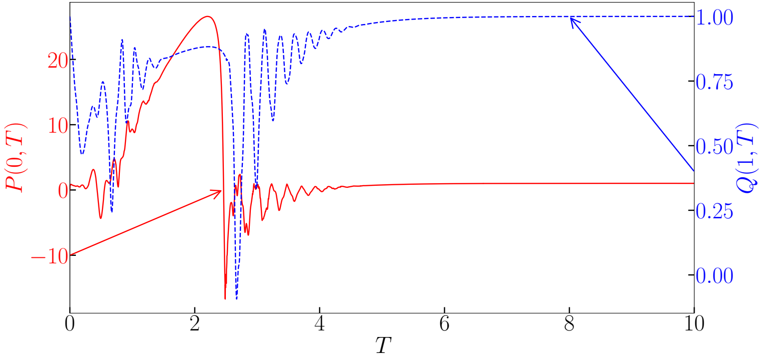




New theory (Wang & Christov, *Proc. R. Soc. A*, 2019) (curves) vs. experimental data from (Kiran Raj *et al.*, *Microfluid. Nanofluid.*, 2017).

[Technical detail: plate theory models fail here due to strong dependence of max $u_y \propto (w/t)^3 \to 0$.]

Transient soft hydraulics: Unsteady fluid–structure interactions



- Use complex transients to enable new modalities of micromixing (low but finite *Re*)?

(Inamdar, Wang & Christov, *Phys. Rev. Fluids*, 2020)

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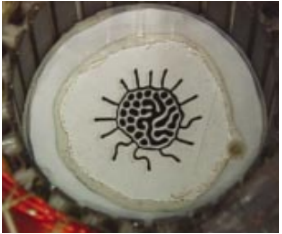
Nonlinear dynamics of confined interfaces

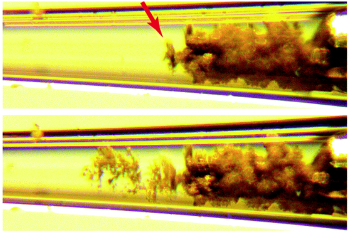
"… the interfaces between two forms of bulk matter are responsible for some of the most unexpected actions. Of course, the border is sometimes frozen (the great Chinese wall). But in many areas, the overlap region is mobile, diffuse, and active … "

(P. G. de Gennes, *Dirac Memorial Lecture*, 1994)



(a) Ferrofluid FSM (Lorenz *et al.*, *PoF*, 2003)

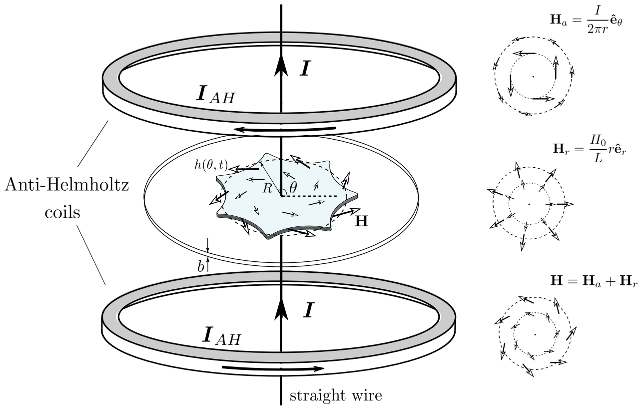
(b) Miscible fingers (Jha *et al.*, *PRL*, 2011)



(c) Dendrites (Bai *et al.*, *Energy Env. Sci.*, 2016)

Video from https://www.youtube.com/watch?v=sBr5fcHILLM

Tuning a magnetic field to generate controllable ferrofluid droplet spin



Hele-Shaw equations for a confined droplet:

$$\boldsymbol{v} = -\frac{b^2}{12\eta}\boldsymbol{\nabla}\left(p - \tfrac{1}{2}\mu_0\chi|\boldsymbol{H}|^2\right),$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{v} = 0,$$

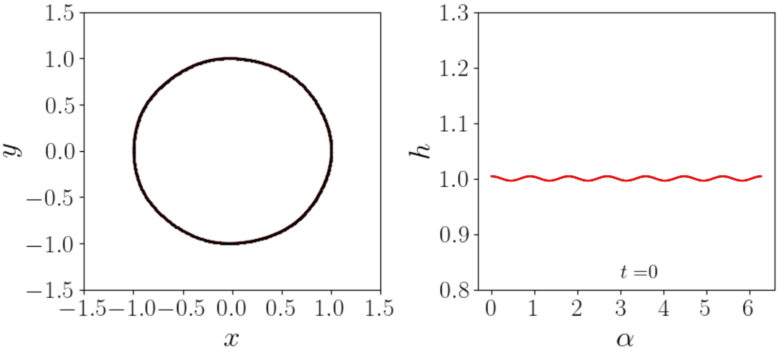
$$p = \tau\kappa - \frac{\mu_0}{2}(\boldsymbol{M}\cdot\hat{\boldsymbol{n}})^2,$$

$$\boldsymbol{M} = \chi\boldsymbol{H},$$

$$\frac{\partial h}{\partial t} = -(\boldsymbol{v}\cdot\hat{\boldsymbol{n}})|\boldsymbol{\nabla} h|.$$

(*e.g.*, Lira & Miranda, *Phys. Rev. E*, 2016)

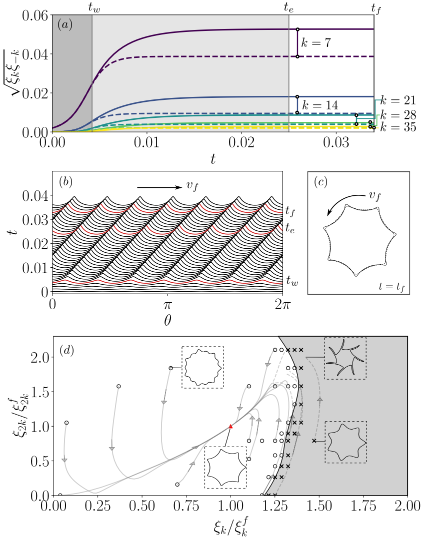
A video is worth 1000 pictures



- Circular interface is linearly unstable ... but weakly-nonlinearly stable.
- Unstable perturbations evolve into a nonlinear interfacial wave
  $\Rightarrow$ circular droplet becomes spinning gear. Why?

(Yu & Christov, arXiv:2009.04644, 2020)

How does nonlinearity arrests linear instability to create a wave?



Weakly nonlinear modal analysis:

$$\dot{\xi}_k = \underbrace{\Lambda(k)\xi_k}_{\text{linear}} + \sum_{k'\neq 0} F(k,k')\xi_{k'}\xi_{k-k'} + G(k,k')\dot{\xi}_{k'}\xi_{k-k'},$$

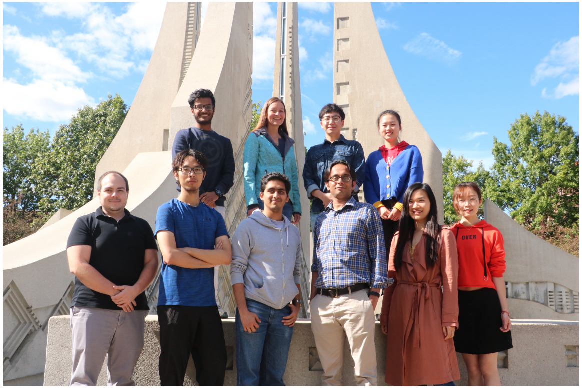
$$\Lambda(k) = \underbrace{\frac{|k|}{R^3}(1-k^2)}_{\text{surface tension}} - \underbrace{\frac{2\mathrm{N_{Ba}}}{R^4}|k|}_{\text{azimuthal } \boldsymbol{H}} + \underbrace{2(1+\chi)\mathrm{N_{Br}}|k|}_{\text{normal } \boldsymbol{H}} - \underbrace{\frac{2\chi\sqrt{\mathrm{N_{Ba}N_{Br}}}}{R^2}ik|k|}_{\text{combined } \boldsymbol{H}},$$

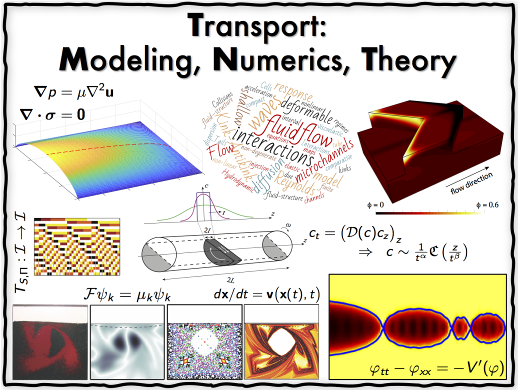
$$k_m = \sqrt{\frac{1}{3}\left[1 - \frac{2\mathrm{N_{Ba}}}{R} + 2(1+\chi)\mathrm{N_{Br}}R^3\right]}.$$

- Our theory suggests how to manipulate a circular droplet into spinning gear.
- We used nonlinear simulation to confirm.

(Yu & Christov, arXiv:2009.04644, 2020)

Thank you for your attention!





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