Towards a theory of soft hydraulics of complex fluids flows through compliant conduits

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Microscale Flows: Mixing and Chemical Reactions & Microscale Flows: Non-Newtonian Fluids (T20.00003)
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What do you picture when you hear “hydraulics”?

Figure: Industrial pipe network, https://www.steeljrv.com/

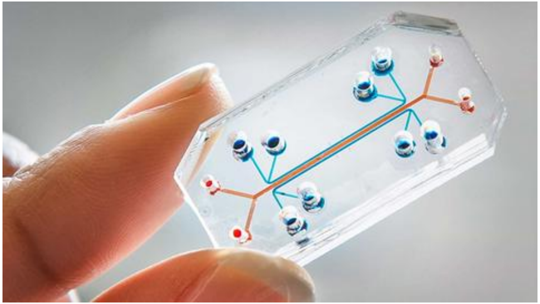


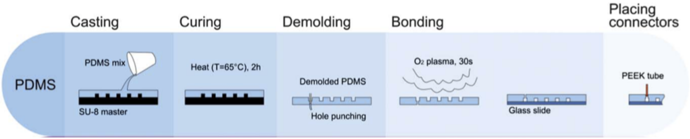
Figure: Microfluidic chip for chemical analysis, https://darwin-microfluidics.com/

▷ Channel diameters ∼ meters, flow ∼ turbulent, materials hard ∼ steel ($E \sim 100$s GPa)

▷ Channel diameters ∼ 100s μm, flow ∼ laminar, materials soft ∼ PDMS (gel, $E \sim 1$ MPa)

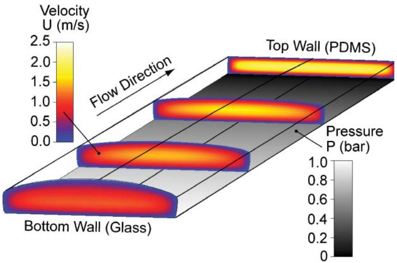
Compliant conduits in microfluidic devices

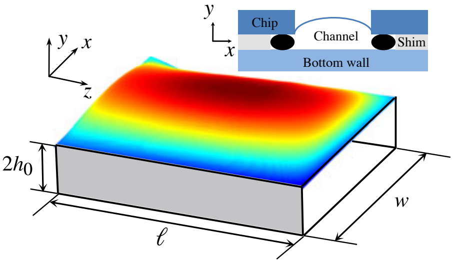
- Lab-on-a-chip fabrication via replica molding/soft lithography. (Sollier *et al.*, *Lab Chip*, 2011)



- Microchannels fabricated from PDMS are soft ⇒ deform due to flow.

(Gervais *et al.*, *Lab Chip*, 2006) (Ozsun *et al.*, *JFM*, 2013)





Big picture: internal flow fluid–structure interactions (FSIs)

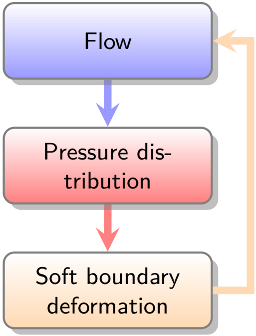


Figure: FSI feedback mechanism.

(Fung, *Biomechanics*, 1997).

- Need ① deformation–pressure (u–p) to determine change in cross-sectional area.

- Need ② the flow rate–pressure grad. relation:

$$q = \left(-\frac{\mathrm{d}p}{\mathrm{d}z}\right)\frac{1}{\mu}\mathfrak{F}\big(\text{geometry}, \underbrace{p}_{\text{FSI}}, \underbrace{\mathrm{d}p/\mathrm{d}z}_{\text{non-Newt.}}\big).$$

  Update classical result (Rubinow & Keller, *J. Theor. Biol.*, 1972).

- The soft hydraulics problem is finding the relationship $\mathfrak{f}(\Delta p, q) = 0$;

  (Then, $R_h \sim -\frac{\partial\mathfrak{f}/\partial q}{\partial\mathfrak{f}/\partial\Delta p}$?)

1. Deformation–pressure relations

- 2D planar, vertical deformation: $u_y(z) = \frac{t}{2G+\lambda} p(z)$

  (*e.g.*, Skotheim & Mahadevan, *PRL* 2004; Chakraborty & Chakraborty, *PoF*, 2010-11)

- 3D axisym. inclusion, radial deformation: $u_r(z) = \frac{a}{4G} p(z)$

  (*e.g.*, Raj M *et al.*, *Biomicrofluidics*, 2018)

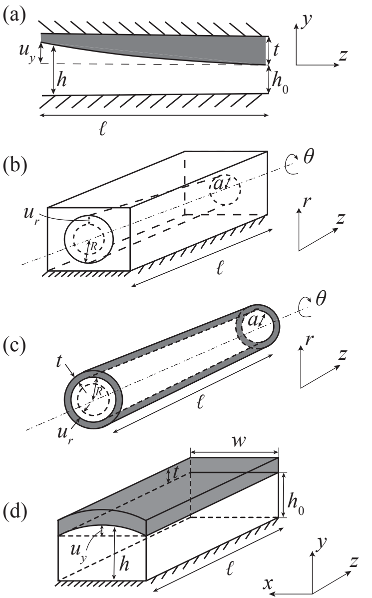
- 3D axisym. shell, radial deformation: $u_r(z) = \frac{a^2}{t\overline{E}} p(z)$

  $\overline{E} = E/(1-\nu_s^2)$ (*e.g.*, Anand & Christov, *ZAMM*, 2021)

- 3D Cartesian, vertical deformation, our approach:

  $$u_y(x,z) = \frac{F(x)}{\overline{E}} p(z) \quad \boxed{\Rightarrow \ \langle u_y \rangle_x(z) = \frac{\alpha w}{\overline{E}} p(z)}.$$

  (Christov *et al.*, *JFM*, 2018; Shidhore & Christov, *JPCM*, 2018; Wang & Christov, *PRSA*, 2019; Anand *et al.*, *ASME JAM*, 2020; Wang & Christov, *PoF*, 2021)



2. Flow rate–pressure gradient relations

- Long and slender geometry ⇒ lubrication approximation. Momentum equation becomes:

$$\boldsymbol{\nabla}_\perp \cdot [\eta(\dot{\gamma})\boldsymbol{\nabla}_\perp v_z] = \frac{\mathrm{d}p}{\mathrm{d}z}, \qquad (*)$$

$\boldsymbol{\nabla}_\perp$ is in the cross-sectional (x, y) or (r, θ) coords; flow is ≈ unidirectional in z, varying slowly.

- Flow rate is

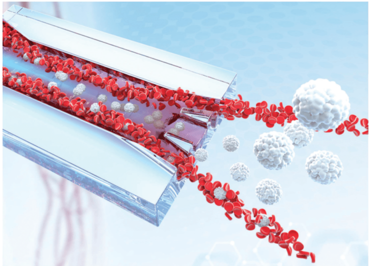
$$q = \iint_{\mathcal{A}} v_z \,\mathrm{d}A. \qquad (**)$$

- Using solution for v_z from $(*)$ into $(**)$, obtain:

$$-\frac{\mathrm{d}p}{\mathrm{d}z}\mathfrak{G}\left(p, \frac{\mathrm{d}p}{\mathrm{d}z}\right) = q,$$

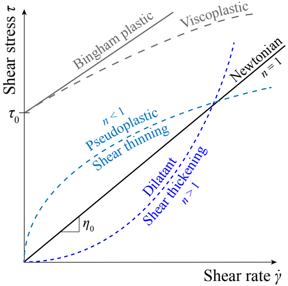
the sought-after $\boxed{\text{ODE for } p = p(z)}$, . . . <u>if</u> we could find analytical v_z expression. ⚠

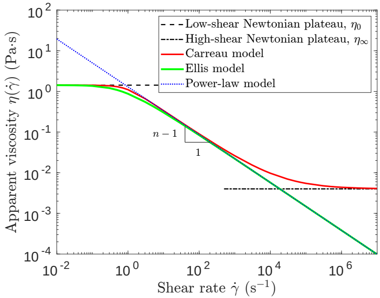
Shear-dependent rheology of complex/bio fluids



www.bioanalysis-zone.com/five-microfluidic-device_loc

(Zhou & Papautsky, *Lab Chip*, 2019)





(Christov, *JPCM*, 2021; fluid properties from Boger, *Nature*, 1977)

$$\tau/\dot{\gamma} = \eta(\dot{\gamma}) = \begin{cases} K|\dot{\gamma}|^{n-1}, & \text{Ostwald–de Waele (power-law);} \\ \dfrac{\eta_0}{1+(\tau/\tau_{1/2})^{n_e-1}}, & \text{Ellis.} \end{cases}$$

Can find v_z and q–dp/dz explicitly for these generalized Newtonian fluids!

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A new approach to the non-Newtonian case, allowing analytical result

1. For power-law (similar for Ellis):

$$q = -\frac{dp}{dz}\overbrace{\left[\frac{h^{2+1/n}w}{2^{1+1/n}(2+1/n)K^{1/n}}\left|\frac{dp}{dz}\right|^{(1/n)-1}\right]}^{\mathfrak{G}(p,dp/dz),\ \text{computed}}.$$

2. Span-wise averaged deformation–pressure relation:

$$h = h_0 + \langle u_y\rangle_x(z) = h_0 + \underbrace{\mathcal{C}}_{\text{we calculated}} p(z).$$

3. Using (1) + (2):

$$-\frac{dp}{dz} = \frac{2^{1+n}(2+1/n)^n K q^n}{[h_0+\mathcal{C}p(z)]^{1+2n}w^n},$$

and, voila:

$$\frac{p(z)}{h_0/\mathcal{C}} = \left[1+\hat{n}_c\overbrace{\left(\frac{\mathcal{C}KLq^n}{h_0^{2+2n}w^n}\right)}^{\beta}\left(1-\frac{z}{\ell}\right)\right]^{\frac{1}{2+2n}} - 1.$$

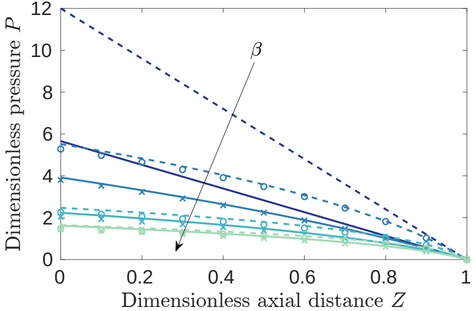

Figure: Dashed, ○: $n=1$; solid, ×: $n=0.5$.

(justification for step (2): Wang & Christov, *PoF*, 2021)

(curves: "full theory" validated to ANSYS, Anand *et al.*, *JNNFM*, 2019)

for convenience: $\hat{n}_c = 2^{1+n}(2+1/n)^n(2+2n)$

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Summary

- The ultimate objective of our research program is to uncover the physics of soft hydraulic systems.
- Introduced the building blocks for predictive theories for flow of complex fluids in compliant conduits.
  - Took into account shear-dependent rheology and different types of wall deformation.

arXiv:2106.07164 [pdf, other]

Soft hydraulics: from Newtonian to complex fluid flows through compliant conduits
Ivan C. Christov
Comments: 30 pages, 13 figures, 258 references; invited topical review for J. Phys.: Condens. Matter
Subjects: **Soft Condensed Matter (cond-mat.soft)**; Fluid Dynamics (physics.flu-dyn)

SPARC Scheme for Promotion of Academic and Research Collaboration

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