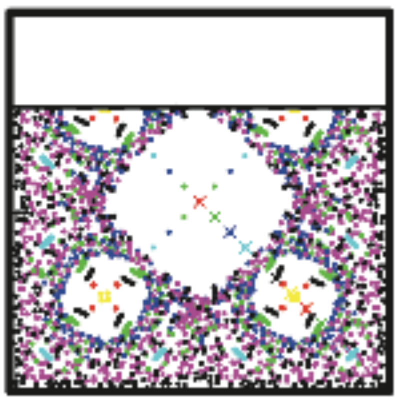
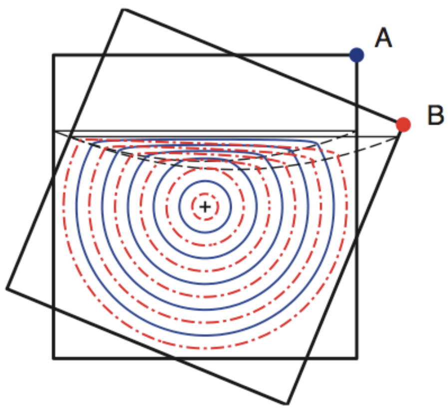
McCormick
Northwestern Engineering

Granular mixing in quasi-two-dimensional tumblers with a vanishing flowing layer



Ivan Christov,
Julio M. Ottino,
Richard M. Lueptow



Robert R. McCormick School of Engineering and Applied Science
Northwestern University

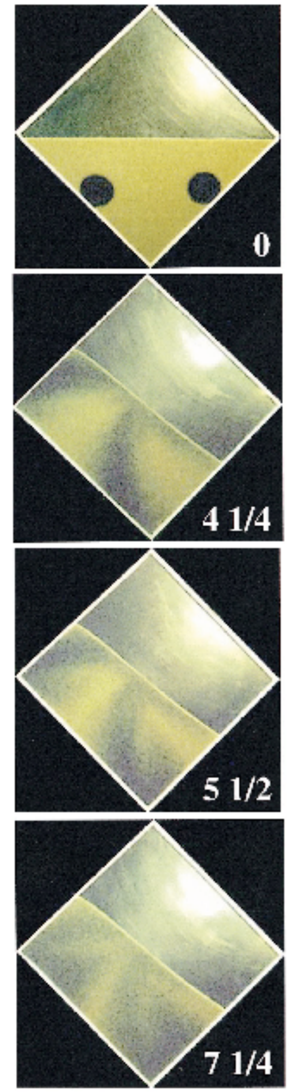
62nd Annual Meeting of the APS DFD
Minneapolis, Minnesota
November 22, 2009

McCormick

Northwestern Engineering

Introduction

- Flowing granular matter is a complex system far from equilibrium.
- A general theory hasn't emerged but mixing in the rolling/continuous flow regime can be understood in terms of geometry and kinematics.
  - Dynamical systems framework exists: can use concepts such as hyperbolic vs. elliptic periodic points, Poincaré sections, etc.
  - Paradigm of mixing by chaotic advection (from fluids) applies.
- But, chaotic mixing of monodisperse granular matter is fundamentally different from fluid mixing.
  - Streamline crossing vs. streamline jumping.
  - Linked twist maps vs. piecewise isometries.

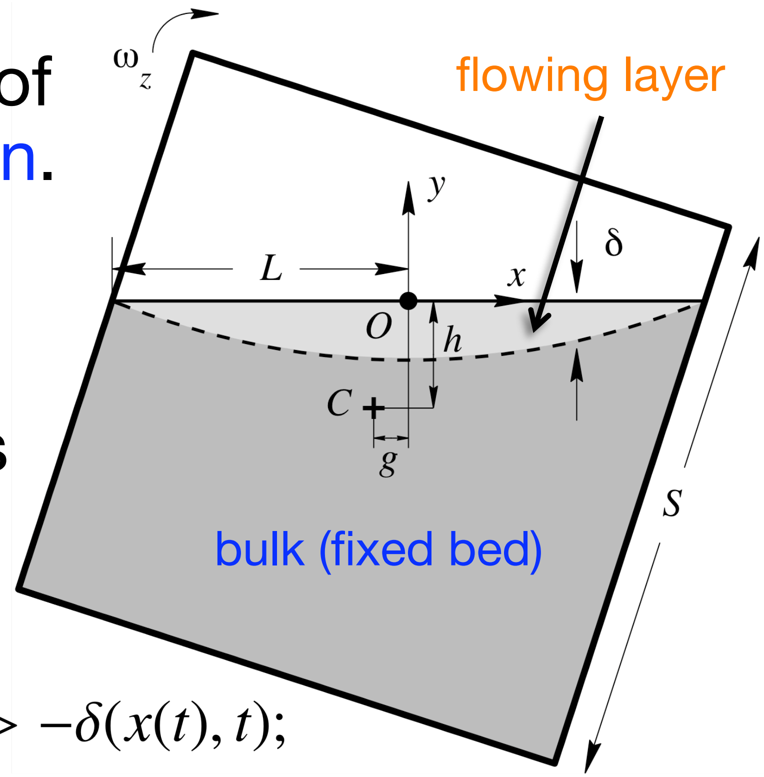


Khakhar et al. 1999

McCormick
Northwestern Engineering

Quasi-2D Tumbled Granular Flows

- Can identify two distinct modes of flow: simple shear & solid rotation.
- Flowing layer translates in time.
- Under a continuum description, focusing on the kinematics leads to a dynamical system for the pathlines in the flow:



$$\frac{\mathrm{d}}{\mathrm{d}t}x(t) = \begin{cases} v_x(x(t), y(t), t), & y(t) > -\delta(x(t), t); \\ \omega_z[y(t) + h(t)] - \dot{g}(t), & \text{otherwise.} \end{cases}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}y(t) = \begin{cases} v_y(x(t), y(t), t), & y(t) > -\delta(x(t), t); \\ -\omega_z[x(t) + g(t)] - \dot{h}(t), & \text{otherwise.} \end{cases}$$

The Basic Model & Assumptions

1. "Bulk" (depth-averaged) streamwise velocity v_x is independent of position. (Khakhar et al. 1999)
   - Streamwise velocity in the flowing layer is linear in depth.
2. Flowing layer adjusts instantaneously to changes in orientation: $\delta_0(t)/L(t) = const. := \epsilon$.
3. And the simplifying assumptions:

$$|\dot{\gamma}_{\text{char}}/\omega_z| \ll 1, \qquad |h(t)/L(t)| \ll 1 \quad \forall t$$

- Balancing the flux in & out of the flowing + using $\nabla \cdot \vec{v} = 0$:

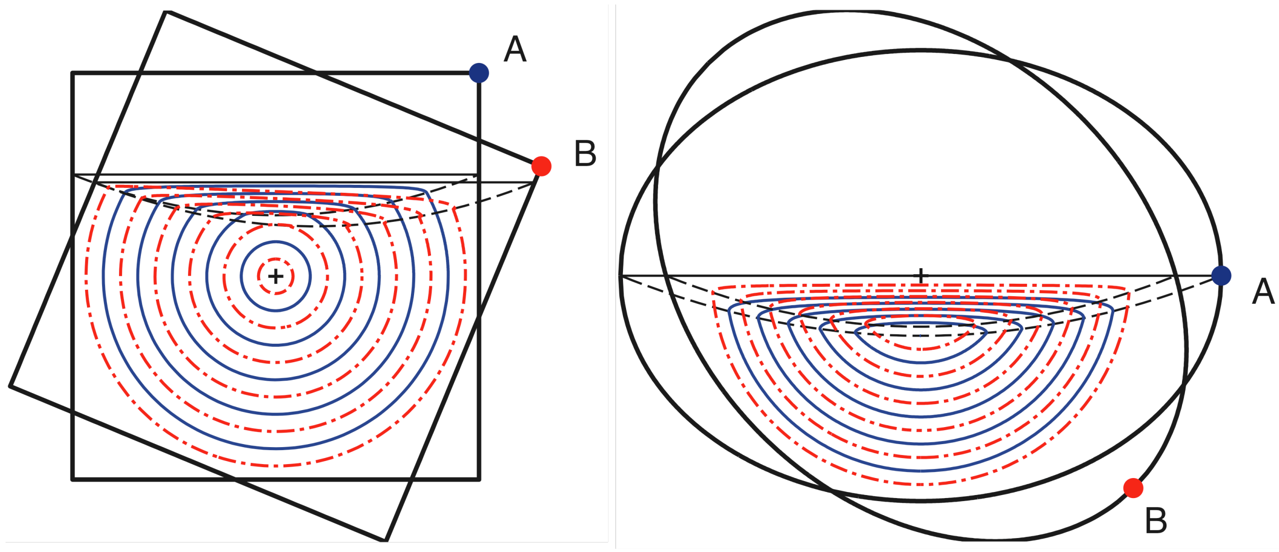
$$v_x(x,y,t) = 2\bar{v}_x(t)\left[1 + \frac{y(t)}{\delta(x,t)}\right],$$

$$v_y(x,y,t) = -\omega_z x(t)\left[\frac{y(t)}{\delta(x,t)}\right]^2$$

$$\delta(x,t) = \delta_0(t)\left\{1 - \left[\frac{x(t)}{L(t)}\right]^2\right\},$$

$$\bar{v}_x(t) = \frac{\omega_z L(t)^2}{2\delta_0(t)}$$

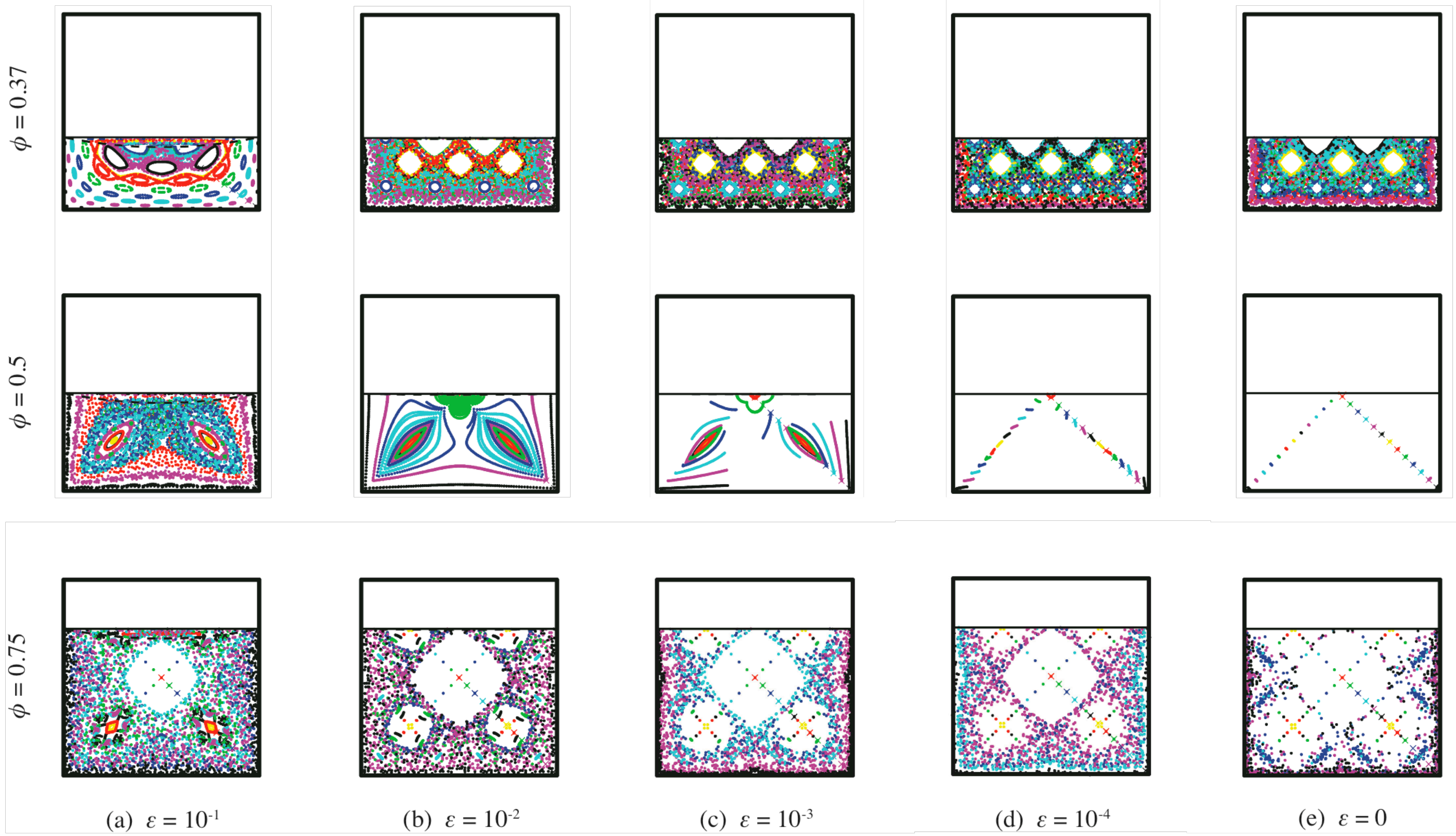
Criterion of Streamline Crossing



(a) 75% full square tumbler (b) 50% full elliptic tumbler

- A (heuristic) sufficient condition for chaotic advection (→ mixing) is that streamlines of the flow at one instant cross those at the following when superimposed. (Ottino 1990, Sturman et al. 2006)
- In tumbled granular flows this can occur only in the flowing layer.

Poincaré Sections and Mixing



The Vanishing Flowing Layer Limit

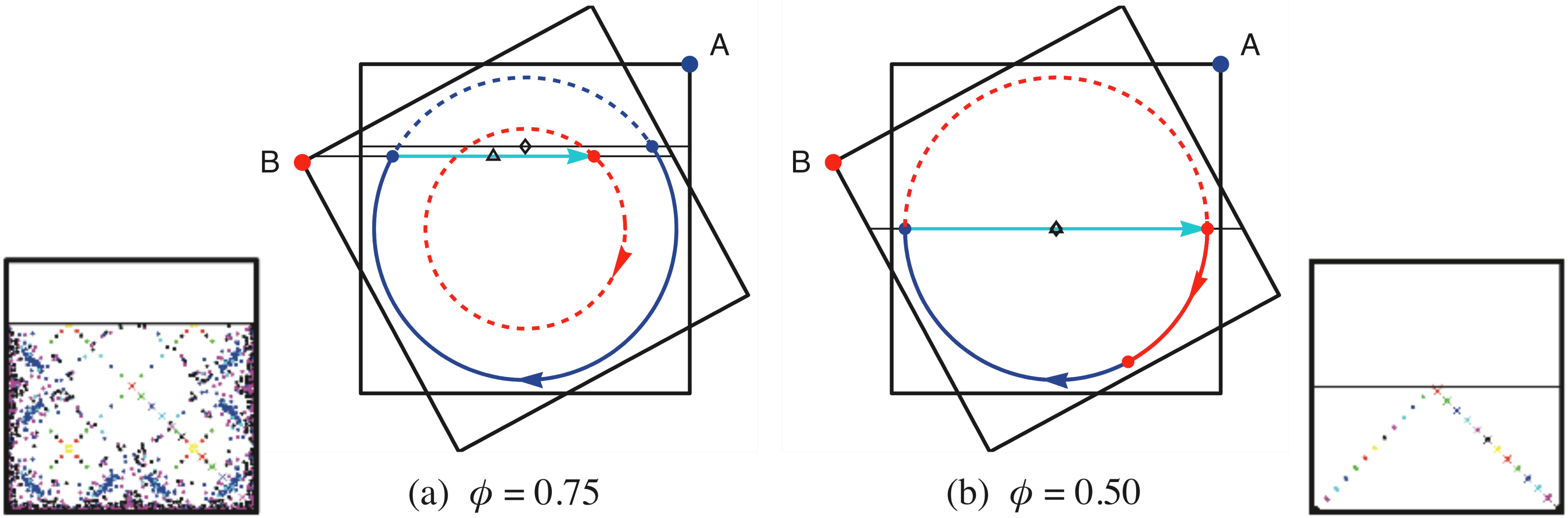
1. As $\epsilon \longrightarrow 0, \quad \delta_0 \longrightarrow 0$ because $L(t)$ is bounded.
   - Thus, the flowing layer becomes an interface collapsing onto the free surface.

2. Surface velocity $\bar{V}_{\text{surf}} = \bar{L}/\epsilon$ becomes infinite.

3. Surface shear rate $\dot{\gamma}_{\text{surf}}(x,t) = \frac{1}{\epsilon^2}\left(\frac{L^2}{L^2 - x^2}\right)$ does too.

4. Since $\delta_0(t) \equiv \epsilon L(t) \approx const. \quad (\epsilon \ll 1)$, assume upon reaching the interface a particle is instantaneously reflected about the midpoint of the flowing layer and exits.

Streamline Jumping Leads to Mixing



(a) $\phi = 0.75$ (b) $\phi = 0.50$

- Key idea: midpoint of the free surface translates in time.
- If a particle reaches the infinitely-thin flowing layer at a time when the midpoint is in a different location, it jumps from one solid body rotation streamline to another.

Piecewise Isometries

- Clearly, when $\epsilon = 0$ the motion is a composition of a rotation $\mathbf{Q}$, a reflection $\mathbf{R}$ and a translation $\mathbf{T}$.
- Each is an isometry. Their composition is also.
- But each time a particle leaves the flowing layer the isometry changes (streamline jumping) → many are joined piecewise: **"cutting & shuffling."**
- It is not hard to put this together to get the map:

$$\mathbf{\Omega}(t_1, t_2) = \mathbf{T}(t_2) \circ \mathbf{R} \circ \mathbf{Q}(t_1, t_2) = \begin{pmatrix} -\cos(\omega_z \bar{t}) & -\sin(\omega_z \bar{t}) & 2g(t_2) \\ -\sin(\omega_z \bar{t}) & \cos(\omega_z \bar{t}) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Summary & Future Work

- ✓ Analyzed the vanishing flowing layer limit of tumbled granular flows and constructed a PWI corresponding to it.
- ✓ Verified that the PWI is indeed the limiting dynamical system, i.e, he “basic state,” of tumbled granular flows.
- ✓ Identified streamline jumping as a new mechanism leading to chaotic dynamics and mixing in tumbled granular flows.

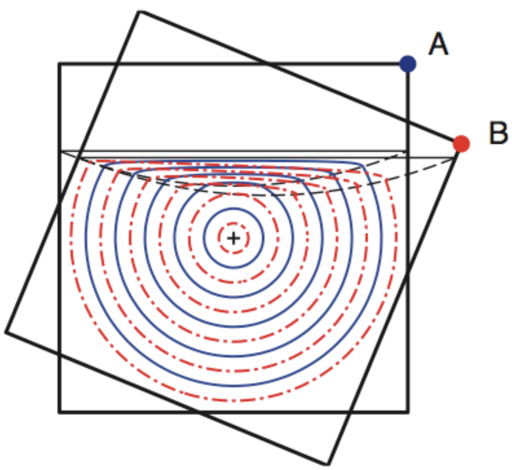
- o Extension to 3D non-circular geometries:
  - o What happen when we compose the streamline-jumping PWI with the multiple-axis-rotation-protocol PWI.
- o Ergodic theory (mathematical mixing properties) of PWIs?
- o Devise a way (or a theorem?) to optimize mixing in a tumbler based on the underlying PWI.
- o Experimental confirmation of the quasi-2D results.

McCormick

Northwestern Engineering

The Laboratory for Complex Systems and Nonlinear Dynamics in Fluids and Granular Materials

Thank you for your attention!



Questions?